

IMAGE CONTRAST ENHANCEMENT USING HISTOGRAM MODIFICATION FRAMEWORK

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Abstract-

The main objective of image enhancement is to improve some characteristic of an image to make it visually better one. A general framework based on histogram equalization for image contrast enhancement is presented. In this framework, contrast enhancement is posed as an optimization problem that minimizes a cost function. Histogram equalization is an effective technique for contrast enhancement. However, a conventional histogram equalization (HE) usually results in excessive contrast enhancement, which in turn gives the processed image an unnatural look and creates visual artefacts. By introducing specifically designed penalty terms, the level of contrast enhancement can be adjusted; noise robustness, white/black stretching and mean-brightness preservation may easily be incorporated into the optimization. Analytic solutions for some of the important criteria are presented.

Keywords- Histogram equalization, histogram modification, image/video quality enhancement.

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I. INTRODUCTION

Image contrast enhancement is an important step in almost every image processing application. The objective of image enhancement is to increase the visual perception of the image so that they are more suitable for human viewers or machine vision applications. It is well known in the image processing society that there is no unifying or general theory for image enhancement algorithms. Thus an enhancement algorithm that is suitable for some application may not work in other applications. This justifies the presence of numerous contrast enhancement algorithms in which can be either application-specific or general algorithms that are proposed to provide the developers of image processing applications with different choices to consider in their applications instead of wasting time in developing new enhancement algorithms. The Contrast Enhancement techniques are used widely in image processing. One of the most popular automatic procedures is histogram equalization (HE) [1], [2].



Fig.1: Image Enhancement

Contrast Enhancement Techniques

Several contrast enhancement techniques have been introduced to improve the contrast of an image. These techniques can be broadly categorized into two groups: direct methods [2], [3] and indirect methods [4], [5]. Direct methods define a contrast measure and try to improve it. Indirect methods, on the other hand, improve the contrast through exploiting the under-utilized regions of the dynamic range without defining a specific contrast term. Most methods in the literature fall into the second group. Indirect methods can further be divided into several

subgroups: i) techniques that decompose an image into high and low frequency signals for manipulation, e.g., homomorphic filtering [6], ii) histogram modification techniques [2]–[7], and iii) transform-based techniques [1]–[5]. Out of these three subgroups, the second subgroup received the most attention due to its straightforward and intuitive implementation qualities.

Contrast enhancement techniques in the second subgroup modify the image through some pixel mapping such that the histogram of the processed image is more spread than that of the original image. Techniques in this subgroup either enhance the contrast globally or locally. If a single mapping derived from the image is used then it is a global method; if the neighbourhood of each pixel is used to obtain a local mapping function then it is a local method. Using a single global mapping cannot (specifically) enhance the local contrast [8], [10]. The method presented in this paper is demonstrated as a global contrast enhancement (GCE) method, and can be extended to local contrast enhancement (LCE) using similar approaches. One of the most popular GCE techniques is histogram equalization (HE). HE is an effective technique to transform a narrow histogram by spreading the gray-level clusters in the histogram [3], [4], and it is adaptive since it is based on the histogram of a given image. However, HE without any modification can result in an excessively enhanced output image for some applications (e.g., display-processing).

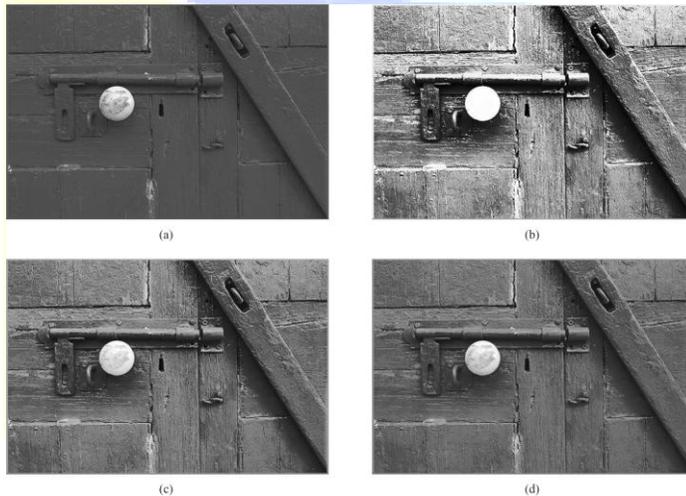


Fig 2: Modified histogram equalization results using (6) for image *Door*. (a) Original image, (b) enhanced image using (6) with $\lambda=0$, (c) enhanced image using (6) with $\lambda=1$, (d) enhanced image using (6) with $\lambda=2$.

II. HISTOGRAM-BASED CONTRAST ENHANCEMENT

This contrast enhancement techniques utilize the image histogram to obtain a single-indexed mapping $T[n]$ to modify the pixel values. In HE and other histogram-based methods, mapping function is obtained from the histogram or the modified histogram, respectively [8]. HE finds a mapping to obtain an image with a histogram that is as close as possible to a uniform distribution to fully exploit the dynamic range. A histogram, $h[n]$, can be regarded as an un-normalized discrete probability mass function of the pixel intensities. The normalized histogram $p[n]$ of an image gives the approximate probability density function (PDF) of its pixel intensities. Then, the approximate cumulative distribution function (CDF), $c[n]$, is obtained from $p[n]$. The mapping function is a scaled version of this CDF. HE uses the image histogram to obtain the mapping function; whereas, other histogram-based methods obtain the mapping function via the modified histogram. The mapping function in the discrete form is given as

$$T[n] = \left\lfloor (2^B - 1) \sum_{j=0}^n p[j] + 0.5 \right\rfloor \quad (1)$$

Where B is the number of bits used to represent the pixel values, and $n \in [0, 2^B - 1]$. Although the histogram of the processed image will be as uniform as possible, it may not be exactly uniform because of the discrete nature of the pixel intensities. It is also possible to enhance the contrast without using the histogram. Black stretching and white stretching are simple but effective techniques used in consumer-grade TV sets [1]. Black stretching makes dark pixels darker, while white stretching makes bright pixels brighter[6]. This produces more natural looking black and white regions; hence, it enhances the contrast of the image. Linear black and white stretching can be achieved by the mapping

$$T[n] = \begin{cases} n \times s_b, & n \leq b \\ n \times g[n], & b < n < w \\ w + (n - w) \times s_w, & w \leq n \end{cases} \quad (2)$$

Where b is the maximum gray-level to be stretched to black and a is the minimum gray-level to be stretched to white, f is any function mapping the intensities in between, and α and β are black and white stretching factors both of which are less than one.

III. HISTOGRAM MODIFICATION

To fully exploit the available dynamic range, HE tries to create a uniformly distributed output histogram by using accumulated histogram as its mapping function. However, HE often produces overly enhanced unnatural looking images. The modified histogram can then be accumulated to map input pixels to output pixels, similar to HE.

The modified histogram can be seen as a solution of a bi-criteria optimization problem. The goal is to find a modified histogram that is closer to u as desired, but also make the residual small. This modified histogram would then be used to obtain the mapping function via (1). This is a bi-criteria optimization problem, and can be formulated as a weighted sum of the two objectives as

$$\min \|h - h_i\| + \lambda \|h - u\| \quad (3)$$

where, h_i = input histogram, h = modified histogram, λ is a problem parameter. As λ varies $[0, \infty)$, the solution of (3) traces the optimal trade-off curve between two objectives. HE obtained by $\lambda=0$ corresponds to the standard HE, and λ as goes to infinity it converges to preserving the original image. Therefore, various

levels of contrast enhancement can be achieved by varying λ .

A. Adjustable Histogram Equalization:-

An analytical solution to (3) can be obtained when the squared sum of the Euclidean norm is used, i.e.,

$$\tilde{h} = \underset{h}{\operatorname{argmin}} \|h - h_i\|^2 + \lambda \|h - u\|^2 \quad (4)$$

which results in quadratic optimization problem

$$\tilde{h} = \underset{h}{\operatorname{argmin}} [(h - h_i)^T (h - h_i) + \lambda (h - u)^T (h - u)] \quad (5)$$

h

The solution of (5) is

$$\tilde{h} = \frac{h_i + \lambda u}{1 + \lambda} = \left(\frac{1}{1 + \lambda}\right) h_i + \left(\frac{\lambda}{1 + \lambda}\right) u \quad (6)$$

The modified histogram \tilde{h} , therefore, turns out to be a weighted average of h_i and u . Simply by changing λ , the level of enhancement can be adjusted instead of the more complex nonlinear technique given by Stark[10].

An example image and enhanced images using modified histogram equalization with three different λ values (0, 1, 2) are shown in Fig. 2. When λ is zero, the modified histogram is equal to the input histogram; hence, the standard HE is applied.

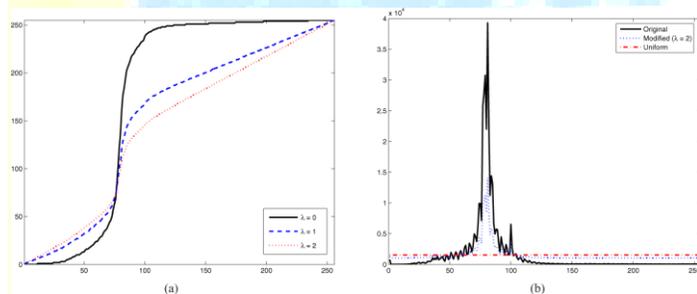


Fig 3. The mappings and histograms for Fig. 1. (a) Mappings for three different λ values used in Fig. 1, (b) original histogram, modified histogram with $\lambda=2$ and the uniform histogram.

The resulting image is over-enhanced, with many unnatural details on the door and loss of details on the doorknob. When λ is increased to one, the penalty term comes into play and the enhanced image looks more like the original image. For $\lambda=2$, the level of enhancement is further decreased and the details on the doorknob are mostly preserved. In Fig. 3(a), the mappings for the three λ values are given. As λ increases, the mapping becomes more similar to $T[n] = n$ line. The fixed point observed around gray-level value of 76 is a repelling fixed point. Although the level of enhancement is decreased with increasing λ , the slope of the mapping at the fixed point, n^* , is still rather large. The slope of n^* at determines how fast the intensities in the enhanced image move

away from the fixed point[3]. This may become especially important for images with smooth background in which gray-level differences in neighbouring pixels look like noise. An example for this situation is shown in Fig. 7(b) and (c).

B. Histogram Smoothing:-

To avoid spikes that lead to strong repelling fixed points, a smoothness constraint can be added to the objective. The backward-difference of the histogram, i.e., $h[i] - h[i-1]$, can be used to measure its smoothness. A smooth modified histogram will tend to have less spikes since they are essentially abrupt changes in the histogram.

The difference matrix $D \in R^{255 \times 256}$ is bi-diagonal

$$D = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

with the additional penalty term for smoothness, the optimal trade-off is obtained by

$$\min \|h - h_i\|^2 + \lambda \|h - u\|^2 + \gamma \|Dh\|^2 \tag{7}$$

solution is

$$\tilde{h} = ((1 + \lambda)I + \gamma D^T D)^{-1} (h_i + h_u) \tag{8}$$

While (6) results in a weighted average of and , (9) further smoothes this weighted average to void spikes. The first term in (9), that is,

$$S^{-1} = ((1 + \lambda)I + \gamma D^T D)^{-1}$$

in fact corresponds to a low-pass filtering operation on the averaged histogram. This can be seen by expressing

$$S = ((1 + \lambda)I + \gamma D^T D)$$

Hence, a penalty term for smoothness corresponds to low-pass filtering the averaged histogram. This shows that the proposed framework provides an explanation for the histogram low-pass filtering approaches investigated as in Gauch’s work [9], from a different perspective.

C. Weighted Histogram Approximation:-

Histogram spikes occur because of the existence of large number of pixels with exactly the same gray-level values as their neighbours. Histogram spikes cause the forward/backward difference of the mapping at that gray-level to be large. This results in an input-output transformation that maps a narrow range of pixel values to a much wider range of pixel values. Hence, it causes contouring and grainy noise type artefacts in uniform regions. A large number of pixels having exactly the same gray-levels are often due to large smooth areas in the image. Hence, the average local variance of all the pixels with the same gray-level can be used to weight the approximation error, $h - h_i$. Histogram approximation error at the corresponding bin will be weighted with a smaller weight. Therefore, the modified histogram bin will not closely follow the input histogram's spike bin to minimize the approximation error[5]-[10]. The objective function with the weighted approximation error is

$$\min(h - h_i)^T W(h - h_i) + \lambda(h - u)^T (h - u) \quad (9)$$

Where $W \in R^{256 \times 256}$ is the diagonal error weight matrix, and $W(i,i)$ measures the average local variance of pixels with gray level. The solution of (9) is

$$\tilde{h} = (W + \lambda I)^{-1}(Wh_i + \lambda u) \quad (10)$$

This is computationally simpler than (8). Since the first term is a diagonal matrix, taking matrix inverse is avoided, i.e., only simple division operations for the diagonal elements are needed to compute its inverse. Fig.4 shows the weighted histogram approximation and histogram smoothing for comparison. The grain-noise-type artefacts around the text are avoided in both methods. The mappings for the two methods is given in Fig.5. The difference of the mapping corresponding to smooth background pixels has further been reduced. However, the mapping is not as smooth as histogram smoothing since no explicit smoothing is performed on the modified histogram.



Fig.4. Comparison results of histogram smoothing and weighted histogram approximation for image *Palermo*. (a) Histogram smoothing using (8) with $\gamma=1000$ and $\lambda=1$, (b) weighted approximation using (10) with $\lambda=1000$.

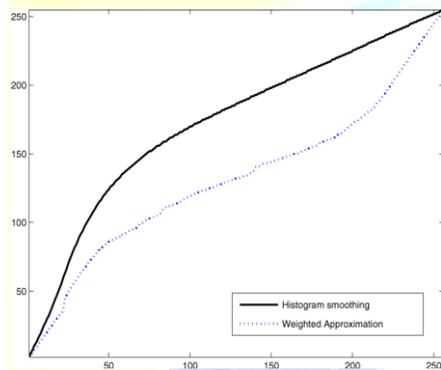


Fig.5. Mappings for the enhanced images given in Fig.4.

D. Black & White Stretching:-

Black and white (B&W) stretching is one of the oldest image enhancement techniques used in television sets. B&W stretching maps predetermined dark and bright intensities to darker and brighter intensities, respectively. To incorporate B&W stretching into histogram modification, where the gray-level range for B&W stretching is $[0, b]$ and $[w, 255]$, respectively, the modified histogram \tilde{h} must have small bin values for the corresponding gray-level ranges. Since the length of the histogram bins determines the contrast between the mapped intensities, by decreasing the histogram bin length for $[0, b]$ and $[w, 255]$, the mapping obtained by accumulating the modified histogram will have a smaller forward/backward difference for these two gray-level ranges[3]-[6]. An additional penalty term for B&W stretching can be added to one of the objective functions presented in previous subsections [e.g., adjustable histogram equalization equation]

$$\min(h - h_i)^T (h - h_i) + \lambda(h - u)^T (h - u) + \alpha h^T I^B h \quad (11)$$

Where I^B is the diagonal matrix. $I^B(i, i) = 1$ for $i \in \{[0, b] \cup [w, 255]\}$, and the remaining diagonal elements are zero. The solution to this minimization problem is

$$\tilde{h} = ((1 + \lambda)I + \alpha I^B)^{-1} (h_i + \lambda u) \quad (12)$$

In Fig. 6, histogram smoothing with and without B&W stretching is illustrated. In this experiment, black stretch gray-level range is $[0, 20]$ and white stretch gray-level range is $[200, 255]$ with α set to 5. With the more natural look of the black and white in the image, the contrast has greatly improved. The mapping as given in Fig. 6(d) clearly shows B&W stretching and the smooth transition to non-stretching region.

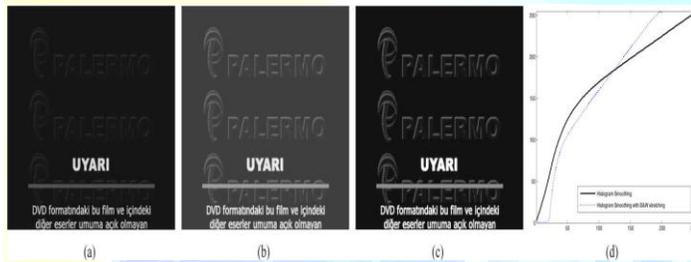


Fig.6. Comparison results of histogram smoothing with and without B&W stretching for image *Palermo*. (a) Original image, (b) enhanced image using (8) with $\gamma = 1000$ and $\lambda=1$, (c) enhanced image using (10) with $\gamma = 100$, $\lambda=1$ and $\alpha=5$ (d) mappings for the two enhanced images in (b) and (c).

IV. QUANTITATIVE MEASURES

Assessment of image enhancement is not an easy task. Although it is desirable to have an objective assessment approach to compare contrast enhancement techniques, unfortunately there is not any accepted objective criterion in the literature that gives meaningful results for every image. There are some metrics used in the literature that approximate an average contrast in the image based on entropy or other measures. If these metrics are used, HE can achieve the best performance even though it may not produce the visually pleasing image, and possibly may produce an un-realistic look. However, it is usually desired to have some quantitative measures in

addition to subjective assessment. Hence, we have the following quantitative measures: Absolute Mean Brightness Error (AMBE), the discrete entropy (H), and the measure of enhancement (EME). AMBE is the Absolute difference between input and output mean. The discrete entropy is used to measure the content of an image, where a higher value indicates an image with richer details. The measure of enhancement (EME) approximates an average contrast in the image by dividing image into non-overlapping blocks, finding a measure based on minimum and maximum intensity values in each block, and averaging them. In addition, a time complexity comparison of HE, weighted threshold HE (WTHE), and the proposed method is included.

A. Subjective Assessment:

i) Gray-Scale Images:

Figs. 7–9 show the original test images and their corresponding contrast enhanced versions. Their mapping functions are shown in Fig. 10(a)–(c), respectively. Hence, images included in this paper are selected among the ones that cause different visual quality. Usually, histogram equalized images result in the best utilization of the dynamic range of the pixel values for maximum contrast. However, this often does not mean that the resulting image is better in terms of visual quality. This situation is also observed with images in Fig. 7(b), and 8(b). Undesired artefacts become more prominent, and amplified nature of noise degrades the quality of the image resulting in an unnatural look. WTHE offers a controllability of the contrast enhancement. Even though WTHE thresholds high and low bin values to prevent its undesired effect, it does not produce as pleasing results as the proposed algorithm does. One other situation HE and WTHE introduces artefacts is when the dynamic range of the original image is shrunk from either one or both ends. In either case, the resulting image is either darkened and/or brightened more than necessary.



Fig.7. Results for image *Plane*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHE

Fig.7(b) is the histogram equalized image of 7(a). HE image, again, looks very unnatural. Especially, the dominance of the sky region results in a very big slope in the mapping function around the pixel value of 250, which results in mapping of range [250, 256] into [150, 256]. Unnatural look of the histogram equalized image is lessened using WTHe. However, it is not alleviated completely. Graininess in the sky still exist in the regions close to the plane.

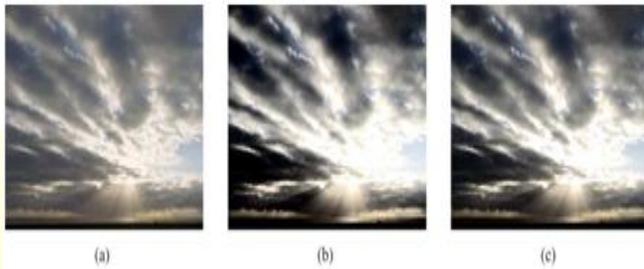


Fig.8. Results for image *clouds*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHe,

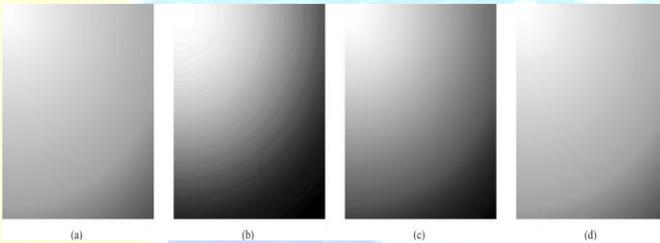


Fig.9. Results for image *nonuniform illumination*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHe,

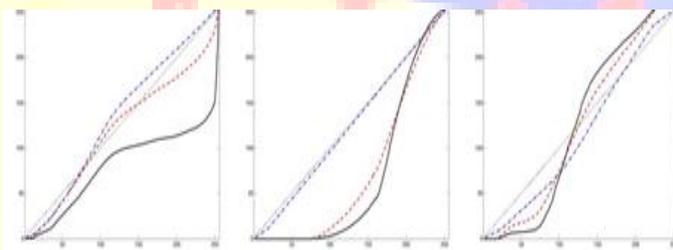


Fig.10. Mappings for enhanced images in Figs. 7, 8 and 9. (a) mappings of Fig. 7, (b) mappings of Fig. 8, and (c) mappings of Fig. 9. Solid line indicates the HE mapping, red dashed line indicates the WTHe mapping, and the dotted line indicates the no change mapping.

Fig. 8(b) is the histogram equalized image of Fig. 8(a). The histogram of the original image occupies bins [75, 255]; as a result, HE results in a darkened image since it stretches the

histogram to increase the dynamic range. A lack of pixel values in the range $[0, 74]$ results in mapping $[75, 255]$ range into $[0, 255]$ range; more specifically $[75, 165]$ range is mapped into $[0, 50]$ and $[165, 220]$ range is mapped into $[50, 220]$. As can be seen from the mapping function in Fig. 10(c), mapping also makes bright regions brighter. One can also observe that HE results in banding. Although the effect of WTHE is not as severe as HE, it also results in darkened image and has slight banding.

ii) Color images

Contrast enhancement can be easily extended to color images. The most obvious way to extend the gray-scale contrast enhancement to color images is to apply the method to luminance component only and to preserve the chrominance components. One can also multiply the chrominance values with the ratio of their input and output luminance values to preserve the hue. Some examples using color images are given in



Fig.9.

Fig.11. Results for image *Hats*. (a) Original image, (b) Enhanced image obtained using HE, (c) Enhanced image obtained using WTHE,

Fig. 9(b) is the histogram equalized image of Fig. 9(a). This image has nonuniform illumination. This becomes more apparent with HE as it stretches the histogram to increase the contrast. The histogram of the original image occupies bins $[17, 233]$. A lack of pixel values in the range $[0, 16]$ and $[234, 255]$ results in mapping $[17, 233]$ range into $[0, 255]$ range; more specifically it darkens the pixels in the range $[17, 118]$ and brightens the pixels in the range $[119, 233]$. One can easily see that the darker clouds become even darker, and clouds in front of the sun become even brighter resulting in loss of details. Although the effect of WTHE is not as severe as HE, it also results in similar artefacts.



Fig.12. Results for image *Window*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHER,



Fig.13. Results for image *Island*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHER,



Fig.14. Results for image *Face*. (a) Original image, (b) enhanced image obtained using HE, (c) enhanced image obtained using WTHER,

B. Objective Assessment

Computed quantitative measures AMBE, and EME listed in Table I supplement the visual assessment presented earlier. Comparison of AMBE values shows that the proposed method outperforms both HE and WTHER in all images except the clouds image. Although HE and WTHER give a smaller AMBE value it does not necessarily mean they are more faithful to the original image. Preserving the mean brightness does not always mean preserving the natural look of an image. HE results in a small AMBE value because HE has an S shaped mapping function. An S

shaped mapping function causes the bright pixels to be even brighter, and dark pixels to be even darker, eventually resulting in a small change in AMBE, although the resulting image has an unnatural look. The same reasoning applies to the image obtained by WTHER. Comparison of H values show that the proposed method performs similar to WTHER and both of them outperform HE. Normally, one would expect HE to give higher discrete entropy value as HE results in more uniform histogram distribution. However, HE results in bin grouping and this decreases the H value. Comparison of EME values show that HE outperforms WTHER. Since EME measures a form of contrast, it is no surprise that HE gives the highest value even though it does not produce the most visually pleasing image.

C. TABLE I: Quantitative Measurement Results

| Image | AMBE | | H | | | EME | | |
|---------------|-------|-------|-------|------|-------|-------|-------|-------|
| | HE | WTHER | Orig. | HE | WTHER | Orig. | HE | WTHER |
| Beach | 34.88 | 24.88 | 6.96 | 5.80 | 6.90 | 5.32 | 32.09 | 12.29 |
| Plane | 48.23 | 5.16 | 6.32 | 4.96 | 6.03 | 9.06 | 24.75 | 12.96 |
| Nonuni. Illu. | 58.61 | 48.23 | 6.92 | 5.93 | 6.90 | 0.16 | 1.44 | 0.58 |
| Clouds | 2.11 | 3.55 | 7.29 | 5.97 | 7.27 | 4.76 | 16.93 | 9.12 |
| Hats | 23.84 | 12.60 | 6.89 | 5.91 | 6.87 | 5.50 | 27.15 | 11.88 |
| Window | 17.11 | 20.84 | 6.80 | 5.79 | 6.82 | 9.11 | 52.32 | 21.01 |
| Island | 22.18 | 9.88 | 7.01 | 5.96 | 6.97 | 8.02 | 51.69 | 20.68 |
| Face | 27.58 | 16.34 | 6.93 | 5.95 | 6.90 | 7.78 | 40.62 | 17.57 |
| Average | 29.32 | 17.88 | 6.89 | 5.78 | 6.83 | 6.21 | 30.87 | 13.26 |

AMBE denotes the Absolute Mean Brightness Error, H denotes the Discrete Entropy, and EME denotes the Measure of Enhancement

V. CONCLUSION

A general framework for image contrast enhancement is presented. The presented framework employs carefully designed penalty terms to adjust the various aspects of contrast enhancement. Hence, the contrast of the image/ video can be improved without introducing visual artifacts that decrease the visual quality of an image and cause it to have an unnatural look. To obtain a real-time implementable algorithm, the proposed method avoids cumbersome calculations and memory-bandwidth consuming operations. Obtained images are visually pleasing, artifact free, and natural looking. A desirable feature of the proposed framework is that it does not introduce flickering, which is crucial for video applications. This is mainly due to the fact that the proposed method uses the input (conditional) histogram, which does not change

significantly within the same scene, as the primary source of information. Then, the proposed method modifies it using linear operations resulting from different cost terms in the objective rather than making algorithmic hard decisions. The proposed method is applicable to a wide variety of images and video sequences. It also offers a level of controllability and adaptively through which different levels of contrast enhancement, from histogram equalization to no contrast enhancement, can be achieved.

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